3.36pt

The Invariant Structure of Equivalent Theories

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The hunt for artifact-free representations

1. Example: coordinate free geometric theories
2. Example: comparativism about quantity

The model isomorphism criterion for theoretical equivalence (IC)

T and T' are equivalent only if their models are isomorphic.

"Hamiltonian and Lagrangian mechanics are not equivalent in terms of statespace structure. This means that they are not equivalent, period." (J. North)

The trouble with IC

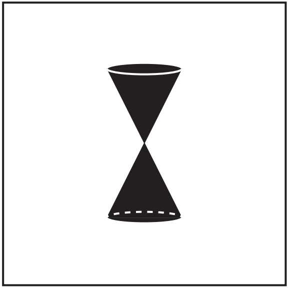
The notion of isomorphism is relative to a species of mathematical structure.

Example

- Linear orderings in signature $\{<\}$.
- Linear orderings in signature $\{\leq\}$.

Example

(1-3) General relativity



(3-1) General relativity

The isomorphism theory of truth (ITT)

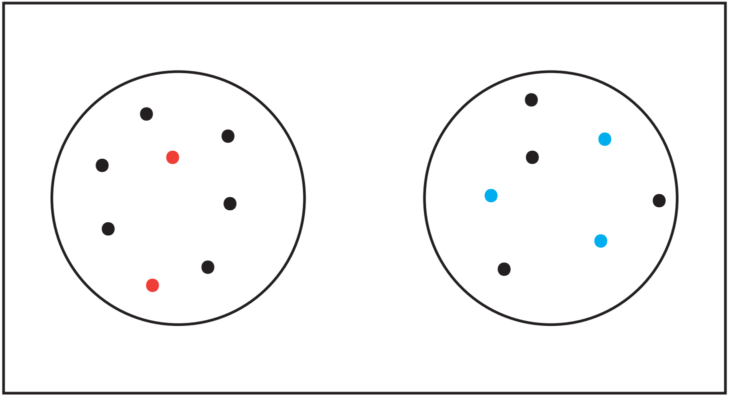
A theory is **true** if the real world itself is isomorphic to one of the theory's models.

The trouble for ITT

$$\mathrm{ITT} \Longrightarrow \mathrm{IC}$$

A **signature** Σ is a set of sort symbols, relation symbols, function symbols, and constant symbols.

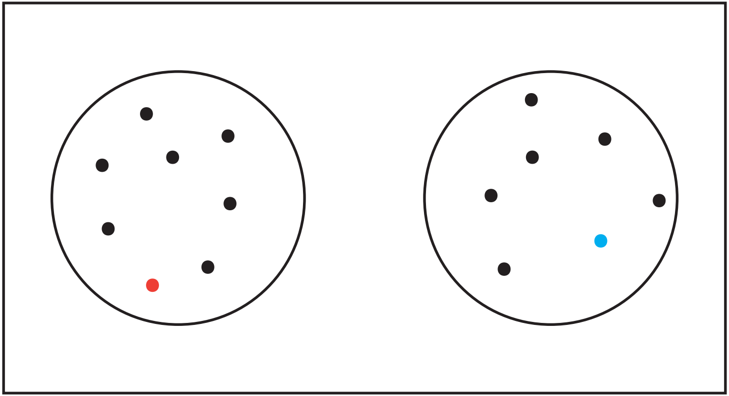
$$\Sigma = \{s_1, s_2, p, q\}$$



A Σ-structure

- there is a unique x of sort s_1 that is p.
- there is a unique y sort s_2 that is q.

A Σ-theory T



A model of the Σ-theory T

Two theories are **logically equivalent** if they have the same class of models.

- there is a unique x of sort s_1 that is p and there is a unique y of sort s_2 that is q.

A theory T' that is logically equivalent to T

There are many pairs of theories that are not logically equivalent, but are nonetheless intuitively equivalent.

- Linear orderings in signature $\{\leq\}$.
- Linear orderings in signature $\{<\}$.

- The theory that says, "there is a p", in the signature $\{p\}$.
- The theory that says, "there is a q", in the signature $\{q\}$.

A **definitional extension** of a theory T is a theory T^+ obtained by adding to T definitions of new predicate symbols, function symbols, and constant symbols.

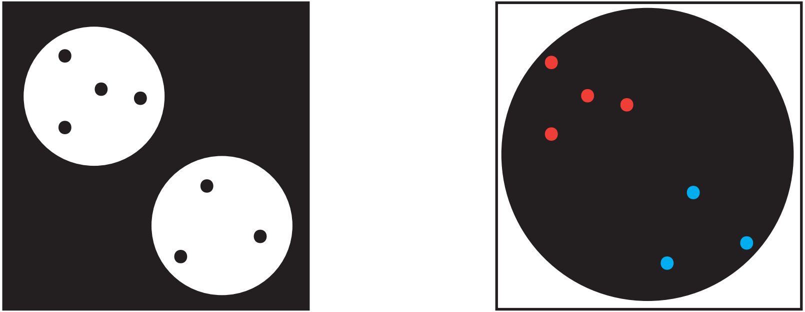
Example: ZFC formalized in $\{\in\}$. Define a relation $\subseteq$ by

$$(x \subseteq y) \leftrightarrow \forall z(z \in x \to z \in y).$$

Two theories are **definitionally equivalent** if they have a common definitional extension.

There are pairs of theories that are not definitionally equivalent, but are nonetheless intuitively equivalent.

- The empty theory in signature $\{s_1, s_2\}$.
- The theory in signature $\{s, p, q\}$ that says there is an x that is p, a y that is q, and every z is either p or q but not both.



models of these two theories

A **Morita extension** of a theory T is a theory T^+ obtained by adding to T definitions of new sort symbols, relation symbols, function symbols, and constant symbols.

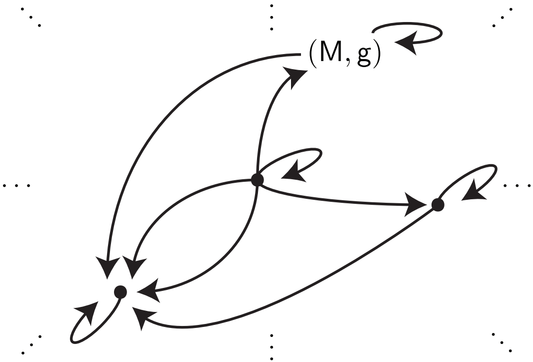
Two theories are **Morita equivalent** if they have a common Morita extension.

Realism without naivete

- $\Sigma = \{s\}$. T says that there are exactly two things.
- $\Sigma = \{s_1, s_2\}$. T' says that there are two things of type s_1, and four things of type s_2. Things of type s_2 are ordered pairs of things of type s_1.

Unfortunately, Morita equivalence doesn't generalize directly to theories beyond the ambit of first-order logic.

- First order theories have categories of models.
- Physical theories have categories of models too.



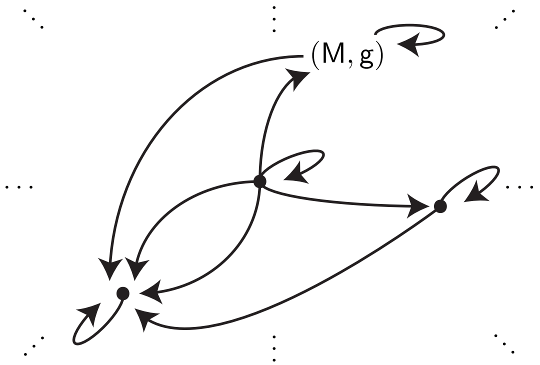
The category of models for general relativity

Two theories are **categorically equivalent** if they have equivalent (structurally identical) categories of models.

Theorem 1. If two theories are Morita equivalent, then they are categorically equivalent.

Theorem 2. There are categorically equivalent theories that are not Morita equivalent.

The disappearance of structure?



The category of models for general relativity