Week 4
Scientific Understanding and Mathematization

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Who am I?

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- Research: philosophy of physics, foundations of quantum mechanics
- Jeg har lært dansk i mange år, men jeg er stadig bedre til engelsk. I må endelig snakke dansk med mig, hvis I vil gøre mig en tjeneste (smiley emoji).

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Today's plan

Part I: Scientific Understanding

- What is scientific understanding, and why does it matter?
- Background: logical positivism, theories of explanation
- De Regt and Dieks: a pluralist account, CUP and CIT

Part II: Mathematization of Physics

- From Aristotle to Newton: the mechanical philosophy
- What did mathematization do to physics? (Gingras)
- Social, epistemological, and ontological consequences

Group Discussion

Think of two examples from your physics education:

1. A phenomenon you feel you genuinely **understand** — not just one you can calculate, but one where you feel you grasp *why* it behaves as it does.
2. A phenomenon you can calculate correctly but do **not** feel you understand.

What is the difference between the two cases?

Possible examples: double-slit interference, quantum tunnelling, entropy increase, orbital mechanics, the photoelectric effect, Bell inequality violations.

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De Regt and Dieks on Quantum Non-Locality

- Henk de Regt (Nijmegen) and Dennis Dieks (Utrecht) are philosophers of science from the Netherlands. They have made central contributions to discussions about the foundations of quantum mechanics.
- De Regt and Dieks have wrestled with the apparent unintelligibility of quantum non-locality.
    - Some interpretations of QM try to make non-locality intelligible by appeal to superluminal causation.

Henk de Regt

Dennis Dieks

- De Regt and Dieks offer us a “theory” of what scientific understanding is — or, at least, what its characteristic signs are
- Their account of understanding is a successor to several competing accounts of scientific explanation that were offered between 1960 and 2000
- Their account is intended to show that understanding is **epistemically relevant**

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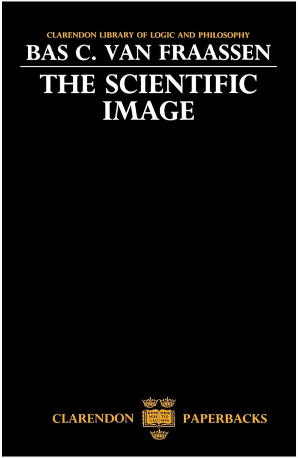
What Drives Science?

Empiricism: The goal of science is to predict the results of experiments.

> *"According to empiricists such as Hempel and van Fraassen, the epistemic aim of science is (roughly stated) the production of factual knowledge about natural phenomena." (p. 141)*

- 1870–1950: Ernst Mach, Logical Positivism
- 1960– : Carl Hempel, Bas van Fraassen, Brad Wray



Bas van Fraassen,

The Scientific Image (1980)

What drives science?

- **Scientific Realism:** Science aims to **explain** phenomena.
  - The realist reaction to logical positivism has been dominant among philosophers in the anglo-american tradition since the 1960s

- Empiricists see the aim of science as knowing **that** while realists see the aim of science as knowing **why**
- Is understanding needed?

What drives science?

- De Regt and Dieks: “We will argue that achieving understanding is among the general (macro-level) aims of science” (p 140)
- “Understanding is an inextricable element of the aims of science” (p 142)

Prediction without understanding?

- **Ptolemaic astronomy**: epicycles predicted planetary positions with impressive accuracy for 1400 years — but provided no physical account of *why* planets move this way
- **Newtonian gravity**: $F = Gm_1 m_2/r^2$ explains why orbits are ellipses and unifies terrestrial and celestial motion — yet Newton's contemporaries objected: what *is* gravity? Action at a distance seems occult
- **QED**: predicts the electron g-factor to 12 decimal places, yet Feynman called renormalization "a dippy process" — "I don't understand it, but I know how to do it"

Is accurate prediction sufficient? Or do we want something more?

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Logical positivism and scientific understanding

- In the early 20th century, the **logical positivists** (Frege, Carnap, the Vienna Circle) set out to put science on a rigorous footing
  - Goal: define all legitimate scientific concepts in terms of logic and observation
  - Anything not so definable was dismissed as *metaphysics* — meaningless
- On this view, "understanding" and "comprehensibility" are merely **psychological** reactions — not objective features of the world
- Therefore: not legitimate goals of science

Carl Hempel

- Carl Hempel (1905–1997) was the dominant figure in post-war philosophy of science
- He argued that *explanation* is an objective, worldly relation between facts — independent of any person
- But *understanding* is merely subjective:

"Such expressions as 'realm of understanding' and 'comprehensible' do not belong to the vocabulary of logic, for they refer to the psychological and pragmatic aspects of explanation." (Hempel)

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Major Accounts of Scientific Explanation

- **Hempel (1940s–60s)**: Deductive-Nomological Model
  - Explanation = logical deduction from laws + initial conditions
  - Criticized for overgenerating (irrelevance) and symmetry
- **Salmon (1970s–80s)**: Causal Models
  - Statistical relevance $\Rightarrow$ causal relevance
  - Explanation = tracing causal/mechanical processes
- **Friedman & Kitcher (1970s–80s)**: Unificationist Accounts
  - Explanation = increased understanding via unification
  - Fewer independent assumptions; general argument patterns

The deductive-nomological account

Hempel: a fact E is explained by being deduced from a covering law L plus initial conditions C:

$$\frac{\begin{array}{l}\text{Initial condition } C\\ \text{Law } L\end{array}}{\text{Explanandum } E}$$

The model both over- and undergenerates:

- *Asymmetry (flagpole):* shadow length and sun angle logically imply flagpole height — but the shadow doesn't explain the pole.
- *Relevance (birth control):* "all males who take birth control pills fail to get pregnant" yields a valid D-N derivation for John Jones — but the pills are irrelevant.

Causal-mechanical account

- A natural fix: require that the explanans is **causally relevant** to the explanandum.
- Wesley Salmon: a scientific explanation describes the causal mechanism that produces the phenomenon.
- This taps into the old tradition of mechanistic explanation (and visualizability).

Unificationist account

"Science advances our understanding of nature by showing us how to derive descriptions of many phenomena, using the same patterns of derivation again and again, and, in demonstrating this, it teaches us how to reduce the number of types of facts we have to accept as ultimate (or brute)." (Philip Kitcher)

Stalemate

- Proved beyond difficult to isolate the core idea of **explanation** that holds throughout all the different sciences
- The methodology of "whatever examples I can remember from when I was a physics student" was recognized as unacceptable

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Against the old critique of understanding

- “The present paper argues, *pace* Newton-Smith, that understanding can play the desired unifying role.” (p 137)
- “Should we rely on the view of practicing scientists. . . ?” (p 138)

- De Regt and Dieks argue that understanding is an **epistemically relevant** concept.
  "We will argue that understanding . . . is epistemically relevant" (p 138)
- De Regt and Dieks argue that understanding transcends individual psychology.
  "We will argue that understanding . . . transcends the domain of individual psychology." (p 138)

Understanding is contextual

- HH: A phenomenon can be contextual and yet be second-order objective
- For example, it is second-order objective that “Oslo is less than 500km from us”
- HH: De Regt and Dieks have not yet shown the sense in which understanding or intelligibility is second-order objective

Intelligibility is contextual: a case study

- In 1926, Heisenberg's **matrix mechanics** and Schrödinger's **wave mechanics** were proved mathematically equivalent — identical empirical predictions
- Yet most physicists found wave mechanics far more **intelligible**:
  - A continuous wave evolving in time felt tractable and visual
  - Matrix algebra felt opaque — hard to "see" what the theory was doing
- By the 1930s, physicists trained in matrix methods found *it* more natural — the asymmetry faded

Same empirical content, different intelligibility — depending on training and context. Exactly what De Regt & Dieks predict.

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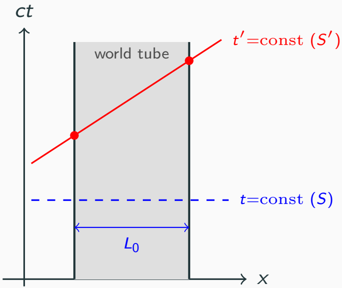
- De Regt and Dieks: Understanding cannot be reduced to any of these particular accounts of scientific explanation
- HH: De Regt and Dieks do *not* say that understanding requires at least one of these types of explanations. This seems like a defect of permissiveness in their account.

- “Salmon treats causality as a standard for intelligibility” (p 144)
- “Present-day scientific developments cast severe doubt on the alleged privileged status of [Salmon’s] model of causal explanation as the way to scientific understanding.” (p 145)
- “At the deepest levels of physical reality Salmon’s concept of causality is highly problematic.” (p 145)
- “Physics is full of examples that show that causal-mechanical explanation is not always the actually preferred manner of achieving understanding.” (p 145)

Quantum mechanics

- “Causal connections of this type . . . do not exist according to quantum theory in its standard interpretation.” (p 145)
    - Arguments against trajectories
    - Bell non-locality

Lorentz contraction



- "The usual way of making the contractions intelligible is by connecting them deductively to the basic postulates of special relativity. ... Causal reasoning is not involved." (p 146)
- More controversial than De Regt and Dieks make it out to be: see Bell, "How to teach special relativity" or H. Brown, *Physical Relativity*

Deflection of light by matter in GTR

"The just-mentioned example undermines the causal conception of understanding, because no causal chains were identified that are responsible for the deflection of the light." (p 157)

- “But even in pre-twentieth-century physics causal-mechanical explanation was not always the norm.” (p 146)
- “Between 1700 and 1850 action-at-a-distance rather than contact action and causal chains dominated the scientific scene.” (p 146)

- “These facts are sufficient to cast doubt on the core idea that causality has a special status as *the* fundamental, privileged standard of intelligibility.” (p 146)
- “It would be erroneous to maintain that visualization is essential for obtaining understanding.” (p 156)
- “The various intelligibility standards proposed by philosophers of science (e.g., visualizability, causality, and continuity) find a place in our approach as ‘tools’ for achieving understanding.” (p 157)

Does causality have a privileged role?

- "By the guidance which analysis in terms of cause and effect has offered in many fields of human knowledge, the principle of causality has even come to stand as the ideal for scientific explanation." (Bohr 1948)
- "The viewpoint of complementarity presents itself as a rational generalization of the very ideal of causality." (Bohr 1948)

- “Unification appears to be an effective tool for achieving understanding, but like causality it is one among a variety of tools.” (p 149)
- “The various intelligibility standards proposed by philosophers of science (e.g., visualizability, causality, and continuity) find a place in our approach as ‘tools’ for achieving understanding.” (p 156–157)

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Criterion for Understanding Phenomena

CUP: A phenomenon P can be **understood** if a theory T of P exists that is intelligible.

Criterion for the Intelligibility of Theories

CIT: A scientific theory T is **intelligible** for scientists (in context C) if they can recognize qualitatively characteristic consequences of T without performing exact calculations.

"What one wants in science is the ability to grasp how the predictions are brought about by the theory." (p 151)

CIT in action: the harmonic oscillator

Why does $V(x) = \frac{1}{2}kx^2$ appear everywhere?

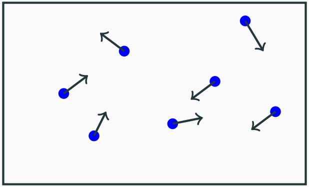
- Pendulum (small angles), LC circuit, molecular vibrations, phonons, modes of the EM field...
- A physicist who has internalized the harmonic oscillator can *qualitatively recognize its consequences without calculation*:
  - resonance at $\omega_0 = \sqrt{k/m}$
  - equally spaced energy levels: $E_n = (n + \frac{1}{2})\hbar\omega$
  - phase-shifted response above and below resonance
- This immediate qualitative grasp, transferable across radically different physical systems, is the mark of a genuinely intelligible theory — CIT in action

Understanding Boyle's Law via the Kinetic Theory i

Illustration of CUP and CIT: How the kinetic theory provides understanding of gas behavior.

- **Qualitative model:** Boltzmann's kinetic theory pictures a gas as a collection of freely moving molecules.
- **Temperature** = average kinetic energy of molecules.
- **Pressure** = cumulative force from molecular collisions with the container walls.

Understanding Boyle's Law via the Kinetic Theory ii



Molecules move randomly and collide with container walls

Understanding Boyle's Law via the Kinetic Theory iii

Key insights (no equations):

- Adding heat $\Rightarrow$ molecules move faster $\Rightarrow$ more forceful, frequent collisions $\Rightarrow$ higher pressure.
- Reducing volume $\Rightarrow$ more collisions per unit area $\Rightarrow$ higher pressure (at constant temperature).

Conclusion: The kinetic theory provides an intelligible, causal-mechanical picture that explains Boyle's law qualitatively — in line with CUP and CIT.

Problems with CIT?

- CIT is ambiguous. Imagine a mechanical arm that pulls slips of paper out of a barrel, and always pulls out correct predictions. Would we be satisfied knowing how the mechanical arm operates?
- It seems that we would want to know how the mechanical arm manages to get the predictions *right* — an explanation not of how it generates predictions, but of why it generates the right ones.

Does QED provide understanding?

- QED predicts the electron's anomalous magnetic moment to 12 significant figures — the most accurate prediction in all of science
- Yet Feynman (one of its architects) was troubled by **renormalization**: divergent integrals are "cancelled" by subtracting infinities
  - Dirac: "I think that the renormalization theory is simply a way to sweep the difficulties. . . under the rug"
  - Feynman: it's a "dippy process" — "I don't understand it, but I know how to do it"
- Does QED satisfy CIT? Physicists learn to use it and recognize its consequences — but many deny they *understand* why it works
- **HH:** a stress test for De Regt & Dieks — is CIT too permissive?

Open questions

- Does the account of De Regt and Dieks have any normative content? Or does it just point out the fairly obvious fact that scientific sub-communities have ideals for understanding?
- They point out that the Copenhagen–Göttingen physicists found matrix mechanics to be intelligible, while most other physicists disagreed. (p 141)

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Part II

Mathematatization of Physics

Group Discussion

Consider the following:

- Newton's law of gravitation, $F = Gm_1 m_2 / r^2$, correctly predicts planetary orbits — but does it *explain* why masses attract? Or does it merely describe the fact that they do?
- Dirac's equation, derived from mathematical requirements of relativistic quantum mechanics, predicted the existence of antimatter before it was observed.

Discussion questions:

1. Is mathematical description the same as physical explanation?
2. Can mathematics itself *lead* us to new physical understanding, or is it only a tool for expressing understanding we already have?

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- In this part, we look at an earlier complaint that mathematization was ruining physics
- Accusation: Isaac Newton is reducing our physical understanding by making everything mathematical

- This case study has wide-ranging implications
  - Interdisciplinarity
    - How can math and physics work together if their goals are different?
    - Are contemporary universities hurting science by building “disciplinary silos”?
  - Good and bad uses of mathematics
  - Methodological pluralism

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Aristotelian physics

- Does <u>not</u> use mathematics or equations
- Predictions are qualitative
  - E.g. rocks tend to fall because they are made of “earthy stuff”
- Physical objects are assumed to have a sort of built-in code that determines their behavior — their **form**

The mechanical philosophy

- The **mechanical philosophy** (Galileo, Descartes) replaced Aristotelian qualitative physics with precise, quantitative description and eliminated **occult qualities**
- Articulated by René Descartes (1596–1650): reduce everything to **configuration** — contact forces, no action at a distance, no force as a fundamental quantity
- Despite using some mathematics, the idea of explanation remains *mechanical*: contact, pushing, pulling

Mathematization of the concept of force

"Though Galileo preceded Newton in applying geometry to free fall, he did not concern himself with the **efficient cause** of that fall. . . The counter-intuitive effects of the mathematization of physical phenomena only began to be perceived with the development of dynamics, that is, that mathematization of the concept of force, as the cause of change in the state of motion." (Gingras, p 387)

$$F = m\ddot{q}$$

"Mathematical explanations came to be preferred to **mechanical explanations** *when the latter did not conform to calculations*." (Gingras, p 398)

"...only true mechanical contact between the parts of a plenum could be considered a *physical* explanation, whereas Newton had simply posited mathematical forces, which pertains to a different order of things, namely geometry, and thus could not constitute a true **explanation** of physical phenomena." (Gingras, p 399)

- Gingras's question: what were the *effects* of mathematization — neither the causes nor the reasons:
  "It is thus the *effects* rather than the causes (or reasons) of the mathematization of physics that I want to follow in this paper." (Gingras, p 384)
- Most interesting for us: the fierce resistance Newton faced. Many physicists argued he would destroy empirical science.
- The philosopher's question — *why* mathematize? — is particularly relevant when considering what we plan to do going forward.

- Gingras sorts the consequences of mathematization into three kinds
    1. **Social**: How mathematization affected people and their relations with each other
    2. **Epistemological**: what kinds of **explanations** were found acceptable
    3. **Ontological**: How mathematization affected physicists' view of what exists

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Social consequences of mathematization

- Summary: The mathematization of physics made understanding its results — and judging them authoritatively — only available to the mathematically adept
- Mathematization put a damper on a tradition of public engagement with physics
  - Not "public" in the sense of the masses, but "public" in the sense of "the wealthy and well educated, but not necessarily in the natural sciences"

- “Euler made plain that the price of entry into the club of legitimate practitioners was an adequate knowledge of mathematics.” (Gingras, p 390)
- “Once the boundaries were well defined and the gate keepers controlled access to the legitimate places of publication, outsiders could make their voices heard only in books or in magazines of general interest.” (Gingras, p 391)

- "It was this very 'right to an opinion' without having the proper prior training that the mathematization of physics was putting into question." (Gingras, p 392)
- "Paralleling the 'rise of public science' … mathematics contributed to the rise of a 'private science', accessible only to the adequately trained." (Gingras, p 393)
- "Mathematization contributed to the formation of a relatively autonomous scientific field, with its control of access mechanisms." (Gingras, p 395)

- “For d’Alembert, the era of verbal (or literary) physics was over; at least in matters concerning the Newtonian world system.” (Gingras, p 393)
- “For them, less talented or less interested in investing long hours in abstruse mathematical calculations, the legitimacy of their verbal contributions to physics was threatened by views like those put forward by d’Alembert.” (Gingras, p 394)

What didn't the mechanical philosophers like about Newton's physics?

- "Privat de Moliéres thus clearly preferred the 'à peu près' of vortex theory to the artificial geometrical precision of Newton's 'metaphysical forces'. His critique of the excessive precision implicit in Newton's mathematization . . ." (Gingras, p 389)
- "This notion that nature does not suffer too much precision, was reminiscent of Aristotle" (Gingras, p 389)

- “Natural philosophy is and ought to be mathematics, that is, the science in which laws relating to quantity are treated according to the principles of accurate reasoning.” (Maxwell 1856)

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- "...it was the very meaning of the term 'explanation' that was at stake in the discussions concerning the legitimacy of hypotheses." (Gingras, p 398)
- Recall that Newton said that he, unlike the Cartesians, didn't make hypotheses
- "This episode shows that the evaluation criteria for what was to count as an acceptable 'explanation' (of gravitation in this case) were shifting towards mathematics and away from mechanical explanations." (Gingras, p 398)

- “Newton was in practice saying that mathematics was replacing verbal formulations as the final arbiter and true explanation of the phenomena.” (Gingras, p 399)
- “Newton had mixed physics with mathematics and thus explained physical phenomena mathematically.” (Gingras, p 399)

“The implicit definition of the term ‘physics’ used here by [Louis Bertrand] Castel is one related to the idea that physics provides mechanical explanations which should not be confused with mathematical explanations.” (Gingras, p 400)

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- **Ontology** is the study of what exists
- Physics surely has a special role in telling us what exists and how it behaves
- Some philosophers believe that physics is the final arbiter of ontology
    - **Reductionism**: the things in other sciences are ultimately just complex versions of the things that appear in physics
    - **Physicalism**: every existing thing is made out of the things described in physics

- One reason some philosophers and physicists find QM unsatisfactory is because they don't think it has a clear ontology
- Is it a fundamental goal of physics to provide ontology?

Physics' standard dilemma about ontology

Plenum views There is "stuff" that fills space

- Aristotle
- Cartesian "extended stuff"
- Wave theories
- Field theories (e.g. electromagnetism)

Particle views There are indivisible chunks of matter separated by empty space

- Lucretius' atoms
- Newton

- “In addition to making possible abstraction and generalization, the manipulation of symbols discussed by Biot and Maxwell were to have an important if often undesired and disturbing ontological effect.” (Gingras, p 404)

Does mathematization obviate ontology?

- Gingras: Mathematization caused physicists to stop thinking about ontology in traditional terms
  - "Only a more detailed analysis could show that the desubstantialization of matter was directly related to the mathematization process itself" (p 406)
- What does he mean by the "desubstantialization of matter"?

- "Mathematics has had the tendency of redirecting the focus of inquiry towards the relational character of the elements, thus contributing to the transformation of concepts and practices." (Gingras, p 406)

Structural realism

- What Gingras notices occurring in attitudes toward ontology is heralded by some physicists and philosophers as a positive movement in the direction of **structuralism**
- Structural realism: physics is not interested in the natures of things, but only in their structural relations with each other

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Mathematization in the 19th and 20th centuries

- The mathematical overhead brought in by Newton was nothing in comparison to what happened in the 19th and 20th centuries
- Mathematics became more rigorous but also more abstract and distant from empirical reality
- The development of non-Euclidean geometry undercut the idea that we are certain of the geometric structure of space
- The development of non-commutative matrix algebra was — according to Bohr — what allowed a rational formulation of quantum mechanics

A brief history of mathematization

- Algebraic geometry: Descartes
- Calculus: Newton
- Lagrangian and Hamiltonian mechanics
- Tensors and differential equations: Maxwell
- Non-Euclidean geometry
  - Undermined the idea that we may be certain of the Euclidean nature of the geometry of space
- Riemannian geometry: Einstein
- Wave equations
- Matrices: Jordan, Heisenberg
- Group theory and group representations: Wigner, Gell-Mann

For reflection

- Is what mathematics did to physics in the 17th and 18th centuries similar to what computers (machine learning, AI) are doing to physics now?
- Are we losing **understanding**, or gaining a new kind?
- Should we fight the trend?
- How can we make best use of the trend?

For reflection

- What is the connection between mechanical explanation and the demand for visualizability (anskuelighed)?
- What is the connection between mechanical explanation and understanding?