Phil Physics: Week 2

Maudlin: intro to book

1. Warm up (easy question): who are the bad guys in Maudlin's story? Who are the good guys?

2. When you're going through the text, note words with strong positive or negative connotations. (e.g. in application to a statement or theory, "clear" has a strong positive connotation.) For each such word, write down — in a sentence or two — what you think Maudlin means by it.

3. What does Maudlin mean by saying that quantum mechanics is not a theory? What does he think it takes to be a theory?

Stern-Gerlach experiments

We will begin now to look at the sort of phenomena that are explained by quantum mechanics. We start with a kind of experiment that was conceived by Otto Stern in 1921, and carrier out by Walther Gerlach in 1922. This experiment was supposed to be a crucial test of the new quantum theory, because it predicted something different than classical physics. In particular, QM says that electrons have a quantity called "spin" that has only two possible values "up" and "down."

Here's the setup of the experiment. First of all, we have a source S that is emitting some stuff. The nature of this stuff is not "plain to sight": it's so small that we cannot see whether we have a bunch of particles, or a continuous field. But we do have control over the direction that this stuff travels, and we can see how it interacts with certain measuring devices.

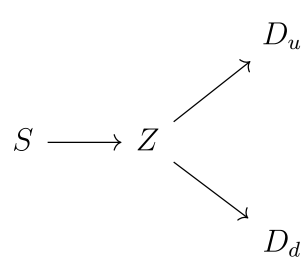
The original Stern-Gerlach experiment involved warming up silver in an oven, which was thought to result in the emission of individual silver atoms, and which were then collimated into a single ray. The important thing about silver atoms is that they have 47 electrons, so the magnetic moments of the first 46 of them cancel each other out. The net magnetic moment of the atom is the same as a single electron. So from now on, we'll speak as the source is emitting a stream of individual electrons. That interpretation is supported by the fact that if the stream is directed to a detector (e.g. a screen), then that detector lights up at discrete moments, and not continuously.

The second thing we have is a pair of magnets that create an inhomogeneous magnetic field between them. When the electron passes through this field, it's motion is expected to be altered by the interaction of its magnetic moment with this magnetic field.

The third thing we have is a screen behind the pair of magnets. When an electron hits this screen, it makes a flash.

The prediction of classical physics was that, because the electrons coming out of the oven have randomly distributed magnetic moments, the flashes on the screen should be uniformly distributed throughout the possible range. The prediction of quantum mechanics was that ...

First experiment



As stated, the result of this first experiment is that D_u registers 50% of the hits, and D_d registers 50% of the hits. While that result was not expected by classical physics, it's not as if it cannot be explained by classical physics.

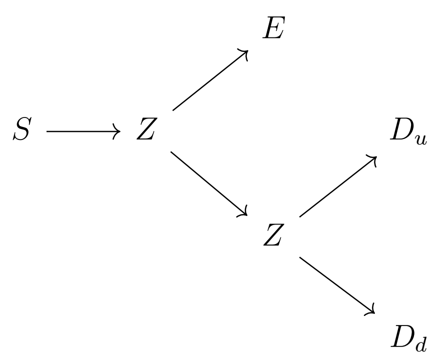
Exercise. Construct a "deterministic hidden variable model" of this experiment.

Solution. Suppose that of the particles emitted by S, 50% are in state Z_+ and 50% are in state Z_-. To elaborate, we suppose that the state space of the system is $\{Z_+, Z_-\}$, and that the quantity Z takes value i in state Z_i. □

Exercise. Suppose that we "look inside" the particles coming out of S, and we can see nothing that explains why some go up and others go down. In other words, we can find no labels like "Z_+" or "Z_-". What kinds of theories could we use to explain the phenomena?

Second experiment

For the second experiment, we replace the detector D_u with an eraser E, and we replace the detector D_d with another Z magnet.



Exercise. What do you predict for the relative number of clicks in D_u and D_d?

If you predicted that D_u and D_d would each get 50% of the clicks, then that goes to show that the results of experiments can be surprising. For, in fact, D_d now clicks 100% of the time.

Exercise. Consider the various explanations we gave for the first experiment. Do any of them fail to explain this second experiment? Does this second experiment give a more clear indication of what electron spin states are like?

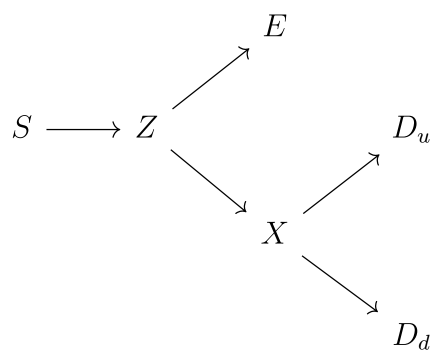
Exercise. Consider the hypothesis:

> (ND) The state of an electron is not disturbed by its passage through Z.

Does this experiment give any positive or negative evidence for ND?

Third experiment

We now consider what happens if we rotate the magnet through 90 degrees. If the previous magnet was called Z, this new one will be called X. In real life, we would now have to think of the experiment as occurring in three-dimensions; but the schematic below projects onto a two-dimensional plane.

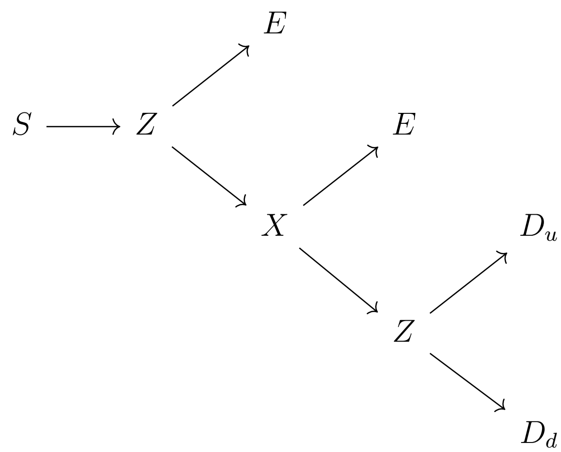


The result of this experiment is that D_u and D_d each register 50% of the hits.

Exercise. Provide a "deterministic hidden variable model" for this experiment.

Fourth experiment

This is where things start getting weird. What we know want to know is what happens if we determine that the electron is Z_-, then we determine that it's X_-, and then we measure Z again. What do you expect the result to be?



The outcome of this experiment is 50% of clicks for both D_u and D_d.

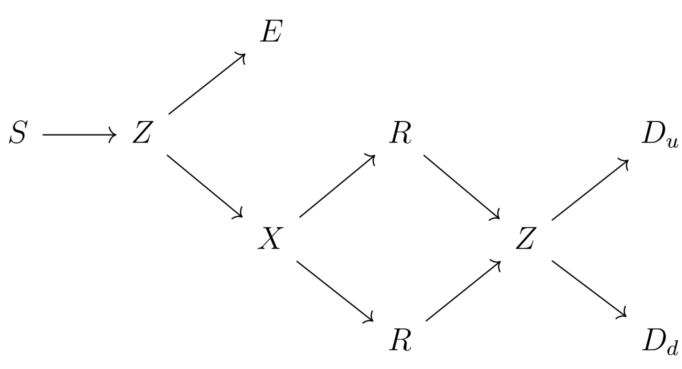
Exercise. What does this experiment say about the non-disturbance (ND) hypothesis?

Exercise. Consider the hypothesis of determinism:

> (D) Each electron has hidden variables X_i and Z_j that determine whether it will go up or down through the various magnets.

Does this experiment rule out D? What about the combination of D and ND?

Fifth experiment



In this more complicated experiment, we have two reflectors labelled with R. When turned on, these reflectors don't do anything besides changing the electron's path. When turned off, these reflectors act like erasers. Here, then, are the results of the experiment:

1. Top reflector off: D_u and D_d register 50%.
2. Bottom reflector off: D_u and D_d register 50%.
3. Both reflectors on: D_d registers 100%.

It seems like something strange is going on here. However, we can modify the experiment to make sure that our Z and X measuring devices are functioning properly. For example, if we replace the reflectors with detectors, then we again get 50% up and 50% down.

Exercise. Consider the hypothesis:

> (EO) Just after the electron goes through the X magnet, it is either 100% on the up path, or 100% on the down path.

What evidence is there that EO is true?

Exercise. Suppose that we begin the experiment without having decided whether to turn one of the reflectors off, and suppose that the electron has just gone through the X magnet. What do you think is the best hypothesis about the location of the electron?

Exercise. Consider the following argument:

> After passing through the X magnet, an electron is either on the top path, or on the bottom path. If it's on the bottom path, then we could turn off the top reflector (without disturbing the electron's state), and then it might end up at D_u. Ditto for the top path. In either case, the electron might end up at D_u.

Do you think this argument is good or bad, and why?

Exercise. Give a single deterministic hidden variable model that explains both the experiment with the top reflector off, and the experiment with both reflectors on.

Solution. Suppose that X magnets cause a particle to spawn a "ghost twin" that goes the opposite direction. (This ghost twin is itself undetectable.) If a particle doesn't meet its twin again, then it behaves no differently than before. If a particle does meet its twin, then it kills him, and changes its own state to Z_-. □

Quantum models of Stern-Gerlach

It's time to see how to use the quantum formalism in order to predict the results of these kinds of experiments.

We assume that each electron has a state v, which is represented by an element of a two-dimensional vector space. For now, it will suffice to think of the most familiar such vector space: the plane $\mathbb{R}^2$ of real numbers.

The following table is an assignment of properties to vectors in $\mathbb{R}^2$.

$$z_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad z_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad x_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

The way we've set things up here, the electron cannot have any of the properties z_i and x_j simultaneously. That's even stronger than the **uncertainty relations**, which would say that you cannot know the values of Z and X simultaneously. So we might do better to talk about the **indeterminacy relations**, since Z and X cannot simultaneously have determinate values.

But what happens if we measure X when the particle is in state z_1 or z_0? First a short answer, then a more elaborate answer. The short answer is that we should think of X as associated with the two states x_0 and x_1; and if we measure X when the system is in state z, then we should:

- Expect 0 with probability $|\langle x_0, z\rangle|^2$, and in this case, change the state to z_0.
- Expect 1 with probability $|\langle x_1, z\rangle|^2$, and in this case, change the state to z_1.

(What we've just written is called **Born's rule**.) Here $\langle x_i, z\rangle$ is the **inner product** of the two vectors x_i and z. In the case at hand, it's none other than $\cos^2(\theta)$, where θ is the angle between x_i and z.

This formalism already predicts the result of the second experiment: after the first measurement of Z, the electrons that go down are in state z_0, and so in the second measurement of Z, they will definitely go down again.

The formalism also predicts the results of the third experiment: after the measurement of Z, the electrons that go down are in state z_0. Hence, a measurement of X should give probability 0.5 for going up, and probability 0.5 for going down.

The fifth experiment is a bit more tricky, and forces us to think harder about what happens at the Stern-Gerlach magnet. Does a X magnet itself cause an electron to "collapse" into either state x_0 or x_1? If that were the case, then we would expect a subsequent measurement of Z to assign equal probabilities to 0 and 1, whereas we know that we will always get 0.

The "orthodox" answer from physics is that a X magnet does not itself cause the state to collapse to x_0 or x_1. What does that is the detectors placed in the two paths after the magnet. Instead, when the electron passes through the X magnet, its spin degree of freedom becomes "entangled" with its spatial location. For simplicity, let's write this as follows:

$$\begin{aligned} x_0 \otimes m &\implies x_0 \otimes d \\ x_1 \otimes m &\implies x_1 \otimes u \end{aligned}$$

(Don't worry if you don't understand the symbols yet; they will be explained.) Then if both reflectors are on, the d and u states transform back to m, which means that the states x_0 and x_1 can again be "superposed" to yield the state z_0.

Superposition

As you know, vectors can be added. The math is straightforward. The physics is also straightforward, if we're talking about **waves**. Just think of two waves approaching the shore from slightly different directions. When they come together, they superpose — at some points, their peaks meet and form a higher wavecrest, and at other points their troughs meet and form a lower depression. Of course, there can also be points where they interfere, or cancel each other out.

We have already represented the state of an electron by a vector. Mathematically, these vectors can be superposed, i.e. added together. But what does that mean physically? Suppose we take the state z_0 where the electron has the property of "down" for Z, and the state z_1 where the electron has the property of "up" for Z, and then we add them together. Does the resulting vector define a physical state, and what is that state like?

Since we have

$$z_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad z_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

[Figure to be supplied in lecture]

it follows that

$$x_0 = \frac{1}{\sqrt{2}}(z_0 + z_1), \qquad x_1 = \frac{1}{\sqrt{2}}(z_0 - z_1).$$

Hence, x_0 is a superposition of z_0 and z_1, and x_1 is a different superposition of z_0 and z_1. That is curious for several different reasons. First, what in the world does Z have to do with X? Aren't these supposed to be independent axes? How could summing a state with one apple and a state with two apples yield a state with one orange? Second, how can summing together states where Z is sharp give rise to states where Z is fuzzy? That's especially puzzling because electrons can't remain ambivalent about which way they'll go through a Z magnet: they have to go up or down.

To get the feel for superposition, it might help to look at another kind of experiment: the famous two-slit interference experiment. Suppose that there's a stream of particles directed toward a screen with two slits, and behind the screen there is another detector screen. Suppose also that there are little doors on the slits that we can open and close.

In the first experiment, we close the bottom door so that the stream only goes through the top door, and we see a pattern of detections on the back screen like this:

That's not surprising: we expect that the particles emerge from the slit with fairly random momentum. What's surprising is what happens when we open the second door. If the source were producing discrete particles, then the prediction of classical physics would be two lumps on the back screen, like this:

In contrast, if the source were producing waves, then classical physics would predict that the waves coming out of the two slits would interfere with each other, producing an interference pattern on the back screen.

Quantum mechanics also predicts the interference pattern, and the explanation goes like this: if only the top slit is open, then it prepares particles in the state z_0. If only the bottom slit is open, then it prepares particles in the state z_1. However, if both slits are open, then the state is $\frac{1}{\sqrt{2}}(z_0 + z_1)$. This latter state is *not* a state in which the particle definitely goes through the top or bottom slit. Instead, it's more like a wave that goes through *both* the top and bottom slits, and then interferes with itself on the other side.