Lecture 23: length contraction

We begin at the popular science level:

> Claim: according to STR, if you are carrying a meter stick and you go very fast, then the meter stick will shrink in length.

Why should you already be suspicious of the way that the claim is put?

Open question (that we will not deal with right now): To what extent has the claim been tested empirically? To what extent can it be tested empirically?

However, there is something correct about the claim. We want to understand (a) what exactly is correct, and (b) what it means vis-a-vis the metaphysics and epistemology of spacetime

As to (b), there are two extremist points of view:

- (Minkowskian) There aren't really any three-dimensional objects. There are four-dimensional objects, and the phenomenon of length contraction is how those four-dimensional objects appear in different reference frames.
- (Lorentzian) Length contraction is a physical effect of one physical substance (the aether) on another physical substance (the material that makes up the rod).

The geometry of length contraction

I will describe length contraction in two different ways. (1) A single measuring rod moving relative to some observer. (2) Two measuring rods in motion relative to each other.

Definition. The *proper length* of a rod is its spatial length in its own reference frame.

Of course, the notion of proper length only makes sense for a rod that is in inertial motion. For a rod that is accelerating, funny stuff can happen.

- There is one sense in which there is *no* length contraction: If we take a rod γ and apply a Lorentz boost L, then the resulting object $L(\gamma)$ has the same spatial length as the original object (see Figure 1). We should not be surprised by this because Lorentz transformations preserve spacetime lengths, and they also preserve the relation "is spacelike related" and the predicate "is a spacelike object". So it is incorrect to say "if γ were moving faster, then it would be shorter."
- There is another sense in which there is length contraction. In particular, if we look at the two-dimensional sheet swept out by a rod γ (moving relative to B), and if we measure the instantaneous length in B's frame of reference, then this instantaneous length will be shorter than $|\gamma|$. To be precise:

Proposition. Let γ be a line segment that is perpendicular to the timelike vector $u \in V$. Let $v \in V$ be a timelike vector, and let $P(\gamma)$ be the projection of γ onto $v^{\perp}$. Then $|P(\gamma)| \leq |\gamma|$, with equality only if v is proportional to u.

This last proposition can be read as saying that if one observer A is carrying an extended object γ, then A's account of the length of γ is expected to be longer than the estimate of an observer B who is in motion relative to A. (For reflection: how might we operationalize length measurement, and why would we expect that observers in relative motion would measure things as having different lengths?)