Lecture 18

Poincaré, On the Foundations of Geometry

1. Advance warning: Poincaré says some things about human perception that may have been empirically falsified. How much of his argument can stand without these assumptions?

2. There are two spaces (sensible and geometric) and they don't look at all like each other

   (a) Sensible space has very little (if any) intrinsic mathematical structure [this is where Poincaré makes empirical assumptions that are questionable].

      i. Sensations have no spatial character (p 117)
      ii. No notion of distance between sensations (p 117)
      iii. No notion of contiguity of sensations

   (b) Geometric space is not a form of intuition, but a cognitive tool for reasoning

   (c) There is no bridge from sensible to geometric space

3. Geometry would mean nothing to creatures that could not move — and more generally, that did not have a will

   (a) We judge two sensations as being of the same thing if we can follow an object with our eye (p 121)

   (b) The distinction between displacement and alteration depends on our ability to move

      i. We consider two displacements to be the same if both can be "undone" by the same internal change
      ii. These (equivalence classes of) displacements form a group; if they didn't, then there could be no geometry

4. Geometry is not our attempt to picture "what is out there"

   (a) Recall the focus on the group of displacements

   (b) The standard of simplicity [under convenience] is not some pre-existing geometric ideal. The displacements of EG form a simpler group than the displacements of LG

5. A fiction of rigid bodies is created by our factorizing changes into (a) displacement and (b) alteration

Einstein, Geometry and Experience

1. The puzzle (p 147): "how is it possible that mathematics, being after all a product of human thought that is independent of experience, is so admirably appropriate to the objects of reality"?

2. Einstein's solution: laws of mathematics (refer to reality $\leftrightarrow$ not certain)

   (a) Axiomatics has succeeded in separating the logical-formal from its objective or intuitive content [de-interpretation, cf. Hilbert]

3. Two interpretations of the axioms of geometry (e.g. "through two points in space there passes one and only one straight line")

   (a) Older interpretation: the axioms are self-evident under the self-evident interpretation of the words contained in them

   (b) Newer interpretation: the axioms are (a) to be taken in a purely formal sense, (b) void of content of intuition or experience, (c) free creations of the human mind, (d) first definte the objects of which geometry treats [implicit definition]

   "...mathematics as such cannot predicate anything about objects of our intuition or real objects" (p 148)

   "...the system of concepts of axiomatic geometry alone cannot make any assertions as to the behavior of real objects of this kind, which we will call rigid bodies" (p 148)

4. Re-interpretation

   "...geometry must be stripped of its merely logical-formal character by assigning to the empty conceptual schemata of axiomatic geometry objects of reality that are capable of being experienced" (p 148)

   "To accomplish this, we need only add the proposition: Solid bodies are related, with respect to their possible relative positions, as are bodies in Euclidean geometry of three dimensions" (p 148)

5. Einstein versus Poincaré

   (a) Einstein: we get conventionalism if we reject the equation (body of axiomatic Euclidean geometry $\approx$ practically rigid body of reality)

   HH: What!? Is E saying that we should assume that our measuring devices satisfy Euclidean laws?