

Logic pset 8

Please answer **ONE** of the following questions. Some of these problems are rather complex, but your answer shouldn't be longer than two pages. All of these are exercises in Chapter 9 of HLW.

1. Prove that the set $\{\neg, \leftrightarrow\}$ is not expressively complete.
2. Prove the steps of the soundness theorem for the $\forall I$ and $\forall E$ rules.
3. Show that if φ is a contingent sentence, then φ has an inconsistent substitution instance.
4. In this exercise, you're asked to show that the RA rule is redundant. We write $\Gamma \succ \varphi$ to indicate that there is a proof of φ from Γ that does *not* use RA. Show that if $\Gamma \vdash \varphi$ then $\Gamma \succ \varphi$.
5. Let's say that a sentence φ of propositional logic is "conjunctive" just in case it contains no connectives besides \wedge and \neg . That is, conjunctive sentences do not contain \vee or \rightarrow . If Γ is a set of conjunctive sentences and φ is a conjunctive sentence, we write $\Gamma \succ \varphi$ to indicate that there is a proof of φ from Γ that uses only DN, RA, $\wedge I$, or $\wedge E$. Show that if $\Gamma \vdash \varphi$ then $\Gamma \succ \varphi$. (You don't need to prove the cases for all inference rules that define \vdash ; a representative sample will suffice.)
6. Give introduction and elimination rules for the nand connective \uparrow . (You may use the symbol \perp for a generic contradiction.) Use these rules to derive the sequent:

$$P \vdash (P \uparrow P) \uparrow (Q \uparrow Q)$$

Prove that your rules are sound relative to the truth-table for \uparrow .