Logic pset 7

Please answer **any three** of the following questions; each is worth six points. Write your answers in your own words, making your reasoning explicit. **Resource:** Chapter 8 of *HLW*

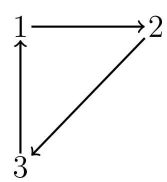
1. Is there a valid proof with the following line fragments? Write your answer in the form of a short essay, using complete sentences.

1	(1)	$\exists x(Fx \to \forall y Gy)$	A
	$\vdots$		
1	(n)	$\exists x Fx \to \forall y Gy$	

2. The sentence $P \to \exists x Fx$ is not existential, and so is not a candidate for EE. But if there is a derivation of φ from $P \to Fa$ and auxiliary assumptions Δ that obeys the restrictions on EE (i.e. the name a doesn't occur anywhere outside of the subproof), then is there also a derivation of φ from $P \to \exists x Fx$ and Δ? Explain your answer.

3. Consider the two sentences $\forall x Fx \to P$ and $\forall x(Fx \to P)$, where x does not occur in P. Are these sentences logically equivalent? Justify your answer by providing proofs and/or models. Write out your answer clearly enough that it would convince somebody who doesn't already get it.

4. Consider the following two interpretations of the binary relation R, one with domain $\{1, 2, 3\}$ and the other with domain $\{\alpha, \beta, \gamma\}$. Write a sentence that is true in one model but false in the other, and explain step by step how to determine its truth value in each. (Note that $1, 2, 3, \alpha, \beta, \gamma$ are elements of the models; they are not names that can be used in your sentence.)



5. For each of the following sentences, provide one interpretation in which it is true and another in which it is false. An interpretation may be presented by giving a set M and a subset R^M of $M \times M$, or it may be presented as an arrow diagram. In either case, explain step-by-step how to determine the truth value of the sentence in the model.

   (a) $\forall x \forall y \exists z(Rxz \land Ryz)$

   (b) $\forall x(\exists y Ryx \to \forall z Rzx)$