logic precept 2

Warmup: Deducing

Exercise 2.5 ($\wedge$E, $\wedge$I, $\vee$I, MP, MT, DN)

$P \to \neg Q, Q \vdash \neg P$

$\neg\neg P \vdash \neg\neg P \wedge (P \vee Q)$

$\neg(P \wedge Q) \to R, \neg R \vdash P$

$\neg P \to \neg Q, Q \vdash P$

$P \vdash \neg\neg(P \vee Q)$

New proof rules

We will work in blocks. The first block leads to problem A1.

$P \to Q \vdash P \to (Q \vee R)$

$(P \vee Q) \to R \vdash P \to R$

$P \to Q \vdash (R \to P) \to (R \to Q)$

$P \to Q \vdash (Q \to R) \to (P \to R)$

(A1) $P \to (Q \to R) \vdash Q \to (P \to R)$

The second block leads to A2. The focus is on the "contrapositive maneuver".

$P \to Q \vdash \neg Q \to \neg P$

$\vdash P \to (P \vee Q)$

$\neg(P \vee Q) \vdash \neg P$

$\vdash (P \wedge Q) \to P$

(A2) $\neg P \vdash \neg(P \wedge Q)$

The third block leads to A3 and A4.

$P \vdash (P \to Q) \to Q$

(A3) $P \vdash (P \to \neg P) \to \neg P$

(A4) $Q \vdash \neg(Q \to \neg Q)$

The fourth block leads to problem B1.

$P \vee Q \vdash Q \vee P$

$P \vee (Q \wedge R) \vdash (P \wedge Q) \vee (P \wedge R)$

(B1) $P \wedge (Q \vee R) \vdash (P \wedge Q) \vee (P \wedge R)$

The fifth block leads to problem B2.

$P, \neg P \vdash Q$

(B2) $P \vee Q, \neg P \vdash Q$

Evaluating proofs

Exercise Which of the following proofs with CP is correct? If a proof is not correct, explain what is wrong with it, and say whether there is a simple fix, or whether it is fatally flawed.

Deps	Line	Formula	Justification
1	(1)	$P \wedge Q$	A
1	(2)	P	1 $\wedge$E
1	(3)	Q	2 $\wedge$E
	(4)	$P \to Q$	2,3 CP

Deps	Line	Formula	Justification
1	(1)	Q	A
2	(2)	P	A
1	(3)	$P \to Q$	2,1 CP

Exercise Explain what is wrong with the following "proof".

Deps	Line	Formula	Justification
1	(1)	$P \vee Q$	A
2	(2)	P	A
3	(3)	Q	A
2,3	(4)	$P \wedge Q$	2,3 $\wedge$I
2,3	(5)	P	4 $\wedge$E
1	(6)	P	1,2,2,3,5 $\vee$E

Additional practice problems

$\neg P \vee \neg Q \dashv\vdash \neg(P \wedge Q)$

$P \to (P \to Q) \vdash P \to Q$

$(P \vee Q) \to R \vdash P \to R$

$P \to (Q \to R), P \to Q \vdash P \to R$

$P \to (Q \to R) \vdash (P \to Q) \to (P \to R)$

$(P \to Q) \to P \vdash (P \to Q) \to Q$

$(P \to Q) \to P \vdash \neg P \to P$

$(P \to R) \wedge (Q \to R) \vdash (P \vee Q) \to R$

$P \vee (Q \vee R) \dashv\vdash (P \vee Q) \vee R$

$P \wedge (Q \vee R) \dashv\vdash (P \wedge Q) \vee (P \wedge R)$

$P \vee (Q \wedge R) \dashv\vdash (P \vee Q) \wedge (P \vee R)$

$\neg P \vee Q \dashv\vdash P \to Q$

$\neg(P \to Q) \dashv\vdash P \wedge \neg Q$

$\vdash (P \to Q) \vee (Q \to P)$

$P \to (Q \vee R) \vdash (P \to Q) \vee (P \to R)$

$\vdash ((P \to Q) \to P) \to P$ (Hint: One possibility is to first prove $\vdash P \vee \neg P$, and then argue by cases. The first case is easy if you remember "positive paradox". For the second case, remember "negative paradox", i.e. that $\neg P$ implies $P \to Q$.)

$P \to (Q \vee R) \vdash \neg R \to (\neg Q \to \neg P)$

$P \to \neg P \dashv\vdash \neg P$

$(P \to Q) \to Q \vdash (Q \to P) \to P$

$(P \to Q) \to R \vdash (P \to R) \to R$

$(P \to R) \to R \dashv\vdash P \vee R$ (Hint: derive $\neg P \to R$ from the sentence on the left.)

$(P \to Q) \to P \dashv\vdash P$ (Hint: assume $\neg P$ and derive $P \to Q$.)