

logic precept 1

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Introduction

We have to get up to speed quickly, which means that the first precept will cover both general concepts, and the first steps into our formal system of logic. This is a lot of ground to cover in 80 minutes!

At the end of this week, you should be comfortable with the various symbols \vee , \wedge , \rightarrow , \neg and how they work to build more complex sentences. You should also be comfortable writing simple proofs that involve the intro and elim rules for \wedge , the intro rule for \vee , modus ponens, and modus tollens.

The first part of the precept is primarily qualitative: we take concepts and words that you already use, and we formulate some more precise definitions. We don't think that we necessarily have the "right" definitions of these concepts, or that everyone understands these concepts in exactly the same way. You might rather think of what we are doing as establishing conventions for a fruitful discussion.

General concepts

- Sentence versus statement versus proposition
 - A sentence need not be a statement
 - A statement can be true or false
 - * Vagueness: Natural language statements can seem sort-of-true and sort-of-false, e.g. "it is cold today". We are operating under the idealization that statements are 100% true or 100% false.
- Argument
 - Definition?
 - Good?
 - * Characteristics

- * Examples
- Bad
 - * Characteristics
 - * Examples
- Abductive
- Deductive
 - * **Valid**
- Circular

Validity is the key concept for this course.

Short answer

1. What are the components of an argument?
2. Which kinds of sentences can be premises or conclusions of an argument?
3. Is a valid argument necessarily a “good” argument? What might a good argument have that a valid argument doesn’t?
4. What’s the point of studying the validity of arguments, as opposed to their goodness?
5. If an argument has true premises and a true conclusion, then is it valid?
6. If you disagree with the conclusion of an argument, might you still say that it’s a valid argument?
7. According to the definitions given in lecture, which of the following sentences makes sense, and which do not?
 - a. That’s a true argument.
 - b. That’s a true statement.
 - c. That’s a valid point.
 - d. That’s a valid argument.
 - e. That’s a valid reason.
8. Give an example of a valid argument with false premises and a true conclusion.
9. Give an example of an invalid argument with true premises and a true conclusion.
10. True or False. Discuss.

- a. If an argument is valid, then you might be able to make it invalid by adding further premises.
- b. You can make an invalid argument valid by removing premises.
- c. If a sentence doesn't follow from another, then its denial must.

11. What is the point of arguments?
12. Is making and evaluating arguments a peculiarly *human* thing?
13. Is there an *objective* difference between good and bad arguments?

Argument identification

For each of the following paragraphs: (a) State whether or not that paragraph contains an argument. (Note: by saying that something is an *argument* is not saying that it is a *good* argument.) (b) If there's an argument, identify its conclusion and premises. (c) If there's no argument, explain what's lacking.

1. Professor Plum was in the drawing room. Miss Scarlet was in the kitchen. The murderer used the knife and the evil act was committed in the hall.
2. If Professor Plum was in the drawing room then Colonel Mustard was the murderer. Professor Plum was in the drawing room. So, Colonel Mustard was the murderer.
3. Every student of logic is wise and knowledgeable. Anyone attempting this exercise is a student of logic. Therefore, anyone attempting this exercise is wise and knowledgeable.
4. I am absolutely sick and tired of getting wet every time it rains. From now on I will never forget to take my umbrella with me in the morning. Even if the weather looks fine when I leave I will certainly make a point of taking that umbrella.
5. All human beings are mortal. So, it stands to reason that Socrates is mortal. After all, he is a human being.
6. Professor Plum was obviously the murderer in this instance. For the murderer used the knife and Professor Plum had the knife. And the murder was committed in the hall and Professor Plum was in the hall.

Charitable reading

In real life, the form of an argument can be hidden by various rhetorical strategies, or simply because it is assumed that all parties would agree to some premise (and so that premise is not stated explicitly). What this means is that a good argument analysis will often involve some charitable re-interpretation of the precise text that you have in front of you.

Translation

A. Which of the following sentences is **atomic** and which is **molecular**? If a sentence is molecular, then which atomic sentences does it contain, and which connectives does it contain? How is it put together?

1. Draco will apologize or not get dessert, but he won't do both.
2. The Quidditch match will be canceled if it rains, unless the field can be magically dried.
3. If Taylor Swift expresses support for environmental policies, then she will also advocate for renewable energy, unless she prioritizes economic concerns.
4. If Elon Musk innovates in electric car technology or develops new space exploration methods, then he will be recognized as a pioneer in technology. However, if he fails to achieve progress in either field, he will not be recognized as such.
5. If it is not sunny then we will not go to the beach.
6. Either it's sunny or we will not go to the beach.
7. We will not go to the beach if it's not sunny.
8. We will go to the beach only if it's sunny.
9. It's being sunny is a necessary but not sufficient condition for our going to the beach.
10. A society does not have free speech unless it allows peaceful protests.
11. Professor Plum is the murderer unless the weapon was a candlestick or the crime occurred in the library.
12. Professor Plum is the murderer only if the weapon was the candlestick and the crime occurred in the library, or the weapon was the dagger and the crime didn't occur in the library.
13. Provided, but only provided, that the French Fleet is sailed forthwith for British harbors, His Majesty's Government give their full consent to an armistice for France.
14. For the tenability of the thesis that mathematics is logic it is not only sufficient but also necessary that all mathematical expressions be capable of definition on the basis solely of logical ones.

B. What are some other sentence connectives in English besides and, or, if-then, not? Which of these other sentence connectives are really just other ways of phrasing the first ones?

First steps in proving with complete rigor

Exercise 2.1 ($\wedge E$, $\wedge I$)

1. Prove that $Q \wedge P$ follows from $P \wedge Q$. That is, write $P \wedge Q$ on line (1), then use the rules (\wedge introduction and elimination) repeatedly until you obtain $Q \wedge P$.
2. Prove that $P \wedge (Q \wedge R)$ follows from $(P \wedge Q) \wedge R$.

We now start using the turnstile symbol \vdash as shorthand for saying that the statement on the right of the turnstile can be derived from the statements on the left of the turnstile. If we ask you to demonstrate that $A_1, \dots, A_n \vdash B$, then you're supposed to begin with A_1, \dots, A_n on the first n lines, then add as many steps as you need to derive B on the last line.

Exercise 2.2 ($\wedge E$, $\wedge I$, $\vee I$)

1. $P \wedge Q \vdash Q \vee R$
2. $P \wedge Q \vdash (P \vee R) \wedge (Q \vee R)$
3. $P \vdash Q \vee (P \vee Q)$

(problem 4 is on the homework)

5. $Q \vdash \neg P \vee Q$
6. $P \vdash P \wedge (P \vee Q)$
7. $P \vdash P \wedge (P \wedge P)$
8. $P \vdash (P \wedge P) \wedge (P \wedge P)$

Exercise 2.3 ($\wedge E$, $\wedge I$, $\vee I$, MP)

(problem 1 is on the homework)

2. $(A \vee B) \rightarrow T, Z \rightarrow A, T \rightarrow W, Z \vdash W$
3. $(A \rightarrow B) \wedge (C \rightarrow A), (C \wedge (W \rightarrow Z)) \wedge W \vdash (B \vee D) \wedge (Z \vee E)$
4. $P \rightarrow (P \rightarrow Q), P \vdash Q$
5. $P \wedge (P \rightarrow Q) \vdash P \wedge Q$
6. $P \rightarrow (Q \rightarrow R), P \wedge Q \vdash R$
7. $P \rightarrow Q, (P \wedge Q) \rightarrow R, P \vdash R$
8. $P \rightarrow (Q \wedge R), (Q \vee A) \rightarrow B, P \vdash B$
9. $(P \vee Q) \rightarrow (A \wedge B), P \vdash A$

Exercise 2.4 ($\wedge E$, $\wedge I$, $\vee I$, MP, MT)

(problem 1 is on the homework)

1. $Q \rightarrow (P \rightarrow R), \neg R \wedge Q \vdash \neg P$
2. $P \rightarrow Q, \neg Q \vdash \neg P \wedge \neg Q$

$$3. P \rightarrow Q, Q \rightarrow R, \neg R \vdash \neg P$$

$$4. P \rightarrow Q, \neg P \rightarrow R, \neg R \vdash Q$$

Exercise 2.5 ($\wedge E$, $\wedge I$, $\vee I$, MP, MT, DN)

$$1. P \wedge (Q \wedge R) \dashv\vdash (P \wedge Q) \wedge R$$

$$2. P \dashv\vdash P \wedge P$$

$$3. P \rightarrow \neg Q, Q \vdash \neg P$$

$$4. \neg \neg P \vdash \neg \neg P \wedge (P \vee Q)$$

$$5. \neg(P \wedge Q) \rightarrow R, \neg R \vdash P$$

$$6. P \rightarrow (Q \wedge R), A \rightarrow \neg R, P \vdash \neg A$$

$$7. \neg P \rightarrow \neg Q, Q \vdash P$$

$$8. P \vdash \neg \neg(P \vee Q)$$

Putting it all together

One might imagine the complete process of argument evaluation as follows:

1. Translate the argument from natural language into symbolic form.
2. Prove that the argument is valid. Or show that it is invalid – something that we don’t yet know how to do.
3. Pass judgment on the original argument.

Nobody really ever does that, and the process would also have several pitfalls. One of the main pitfalls is the “false negative”, i.e. the symbolic argument is invalid, but the original argument is valid. In fact, there are good reasons to think that no rigorous symbolic system could ever capture all the ways in which a natural language argument could be valid. So, in the first place, take that as a caution about the range of our methods.

However, it can still be helpful practice to complete the process described above. The hope is that with sufficient practice, you will more quickly see the logical structure behind arguments you encounter in real life.

Let’s undertake the process on a few arguments.

Argument 1

1. If Donald Trump wins the upcoming election, then certain policies detrimental to international relations will be implemented.

2. If certain policies detrimental to international relations are implemented, there will be an increase in domestic social unrest.
3. Therefore, if Donald Trump wins the upcoming election, there will be an increase in domestic social unrest.

Argument 2

1. Universities should encourage diverse viewpoints and free speech is essential for exposing students to diverse viewpoints.
2. If free speech is essential for exposing students to diverse viewpoints, then universities should have policies that protect free speech.
3. Universities should foster a respectful environment.
4. Universities should have policies that protect free speech and foster a respectful environment.