PHI 201 – Introductory Logic **Final Exam Practice Problems**

1. Are the following true or false?

   (a) If two sentences are TT-equivalent, then there is at least one row in their joint truth table in which they are both true.

   (b) $(\mathsf{P} \to \mathsf{Q}) \to (\mathsf{P} \to \mathsf{R})$ and $\mathsf{P} \to (\mathsf{Q} \to \mathsf{R})$ are TT-equivalent.

   (c) Suppose that there is a perfectly correct proof of P in $\mathcal{F}$, starting with no premises. Then P's truth table has "True" in every row under the main column.

   (d) If P is logically true, then its truth table has at least one "True" in the main column.

   (e) The following sentence is a FO-validity.

$$\mathsf{a} = \mathsf{b} \lor \mathsf{a} \neq \mathsf{c} \lor \mathsf{b} \neq \mathsf{c}$$

2. Write out a full truth table in order to determine whether the following argument is valid or invalid. Show your work.

$(\mathsf{P} \land \neg\mathsf{Q}) \to \mathsf{R}$

$\neg\mathsf{P} \leftrightarrow \mathsf{Q}$

$\mathsf{R} \lor \neg\mathsf{P}$

3. Construct a proof of the following argument. Use only the rules of $\mathcal{F}_T$.

$(\mathsf{F} \land \neg\mathsf{G}) \to (\neg\mathsf{G} \land \mathsf{H})$

$\mathsf{F} \to \neg\mathsf{H}$

$\mathsf{F} \to \mathsf{G}$

4. Translate sentences (a)–(c) into FOL. Use the dictionary provided below.

a = Andrew	$\mathsf{L(x)} \equiv$ x is a letter
s = Sally	$\mathsf{A(x,y)} \equiv$ x is addressed to y
$\mathsf{G(x,y,z)} \equiv$ x gives y to z	$\mathsf{I(x)} \equiv$ x is an invitation
$\mathsf{W(x,y)} \equiv$ x is written by y	

   (a) Andrew gives at least two things to Sally.

   (b) No letter written by Sally is an invitation.

   (c) Some invitations are not addressed to someone.

5. Translate the following FOL sentence into English.

$$\exists x \exists y [x \neq y \land \forall z (z = x \lor z = y)]$$

6. Give a precise statement of the rule ∃ **Elim**.

7. Construct a proof of the following argument.

$$\begin{array}{|l} \\ \hline \neg\exists x F(x) \rightarrow [\forall x(G(x) \rightarrow F(x)) \rightarrow \forall x \neg G(x)] \end{array}$$

8. Complete the following definition.

   Two sentences are *FO-equivalent* if and only if...

9. Are the following two sentences FO-equivalent? Justify your claim.

$$\forall x(F(x) \rightarrow G(x)) \qquad\qquad \forall x F(x) \rightarrow \forall x G(x)$$

10. Is $\exists y \forall x(J(x) \rightarrow K(x, y))$ a FO-consequence of $\forall x(J(x) \rightarrow \exists x K(x, y))$? Justify your claim.