Instructions: Please note that there are questions on the front and back of this page. Write your name, preceptor's name, and pledge* on the exam booklet; and write all of your answers in the exam booklet. When you complete the exam, place your exam booklet in the box at the front of the room, and leave quietly. You have three hours to complete the exam. There are 50 possible points in total.

Definitions [3 points each; 6 points total]

1. Let $A_1, \ldots, A_n, B$ be sentences of predicate logic. What does "$A_1, \ldots, A_n \vdash B$" mean?

2. Complete the following sentence: A predicate logic argument with premises $A_1, \ldots, A_n$ and conclusion B is <u>valid</u> if …

Translation [4 points each; 12 points total]

Translate the following sentences into predicate logic notation. In each case, give a "dictionary" for the predicate and relation symbols that you use in the translation. (We have suggested some predicate and relation symbols at the end of each sentence.)

1. There is a professor who is respected by any student who respects any professor at all. (Px, Sx, Rxy)

2. There is a professor who respects only those students who respect her. (Px, Sx, Rxy)

3. There is no largest prime number. (Lxy, Px, Nx)

Proofs and Counterexamples [20 points total]

1. Prove the following using the basic rules of inference. (You may use Sequent Introduction if and only if you include a proof of the the cited sequents.) [8 points]

$$\vdash (\exists x)(y)(Fy \to Fx)$$

* "I pledge my honor that I have not violated the Honor Code during this examination."

2. Consider the following sentence:

$$(x)(y)[(z)(Rzx \rightarrow Rzy) \leftrightarrow Qxy]$$

This sentence implies one of (i) and (ii) below, but not the other; give a proof (using the basic rules of inference) to show the implication in the one case, and give an interpretation to show the lack of implication in the other case. (For the proof, you may use Sequent Introduction if and only if you include a proof of the the cited sequents.) [12 points]

(i) $(\exists y)(x)Qxy \rightarrow (\exists y)(x)Rxy$

(ii) $(\exists y)(x)Rxy \rightarrow (\exists y)(x)Qxy$

Metatheory [6 points each; 12 points total]

Please complete <u>two</u> of the following three problems. If you give solutions to all three, please clearly designate which problems you want to be graded.

1. State and prove the soundness of Reductio ad Absurdum (RAA) relative to predicate logic interpretations.
2. State the definition of the set of well-formed formulas of <u>propositional</u> logic (i.e. what we covered before the midterm exam). Please use the *strict* definition of wffs where "$P \& Q$" is not a wff, but "$(P \& Q)$" is a wff. Prove that every wff has the same number of left parentheses as right parentheses.
3. True or False (explain your answer): If a propositional logic argument is semantically valid (i.e. $X_1, \ldots, X_n \models Y$), and if all occurrences of an atomic sentence A in the argument are replaced with some well formed formula B, then the resulting argument is also semantically valid.

– THE END –