

1. There are more than two continents.

$$\exists x \exists x \exists z (Cx \wedge Cy \wedge Cz \wedge x \neq y \wedge x \neq z \wedge y \neq z)$$

2. Only students who take logic know how to prove everything.

$$\forall x (Px \rightarrow (Sx \wedge Lx))$$

3. All writers like what they write

$$\forall x \forall y (Wxy \rightarrow Lxy)$$

4. Proof by induction: For the base cases, we need to show that $p \wedge q \vdash p$ and $p \wedge q \vdash q$. Those both follow by \wedge elimination. For the inductive case, we need to show that if $p \wedge q \vdash \phi$ and $p \wedge q \vdash \psi$ then $p \wedge q \vdash \phi \rightarrow \psi$. However, if $p \wedge q \vdash \psi$, then there is a proof that begins with $p \wedge q$ and ends with ψ . We can extend this proof by assuming ϕ , and then doing CP. The result would be a proof that begins with $p \wedge q$ and ends with $\phi \vdash \psi$. Therefore $p \wedge q \vdash \phi \rightarrow \psi$. By induction, it follows that $p \wedge q \vdash \phi$ for all $\phi \in \Sigma$.