PHI 201, Final Exam **Spring 2018**

1. Translate the following into predicate logic. You can assume that the domain is people, and so you don't need an additional predicate symbol for "x is a person".

   (a) There is a person who loves all people who love her. (Use Lxy for "x loves y".)
   $\exists x \forall y(Lyx \to Lxy)$

   (b) Every lover loves herself. (A "lover" is somebody who loves at least one person.)
   $\forall x(\exists y Lxy \to Lxx)$

   (c) There are exactly two people.
   $\exists x \exists y(x \neq y \wedge \forall z(z = x \vee z = y))$

2. Could the following sentence be true? Explain your answer.

$$(\neg P \vee Q) \wedge ((Q \to (\neg R \wedge \neg P)) \wedge (P \vee R))$$

   Yes. Let $v(P) = 0, v(Q) = 0$ and $v(R) = 1$. Then $v(\neg P \vee Q) = 1$ and $v(Q \to (\neg R \wedge \neg P)) = 1$ and $v(P \vee R) = 1$.

3. Explain what's wrong with the following attempted proof:

1	(1)	Fa	A
	(2)	$Fa \to Fa$	1,1 CP
	(3)	$\forall y(Fy \to Fa)$	2 UI
	(4)	$\exists x \forall y(Fy \to Fx)$	3 EI

   Line 3: the application of UI is mistaken, because UI must replace all occurrences of the name (a in this case).

4. Prove the following sequent. You can use cut/replacement, but only if you prove the relevant sequents in your exam booklet, and clearly cross-reference them.

$$\vdash \exists x \forall y(Fy \to Fx)$$

5. Give a rigorous, but informal, proof of the following fact of set theory:

$$C - (A \cap B) \subseteq (C - A) \cup (C - B)$$

Here we use the definition:

$$\forall x((x \in (C - X)) \leftrightarrow (x \in C \wedge x \notin X)).$$

Proof: We need to show that every element in $C - (A \cap B)$ is also in $(C - A) \cup (C - B)$. That's the same thing as showing that every element in $C - (A \cap B)$ that is not in $C - A$ is in $C - B$. Furthermore, an element of C is not in $C - A$ just in case it is in A. So we're going to show: if $a \in C - (A \cap B)$ and $a \in A$, then $a \notin B$.

Suppose that $a \in C - (A \cap B)$ and $a \in A$. Then $a \in C$ and $a \notin A \cap B$. By the definition of intersection, $\forall x(x \in A \cap B \leftrightarrow (x \in A \wedge x \in B))$. Since $a \notin A \cap B$, it follows that $\neg(a \in A \wedge a \in B)$. Moreover, since $a \in A$, it follows that $a \notin B$, which is what we wanted to prove.

6. Provide a countermodel to show that the sentence on the left does *not* imply the sentence on the right. In your countermodel, you should explicitly specify a domain, and extensions for all the predicate symbols.

$$\exists x(Fx \rightarrow \exists y Gy) \qquad\qquad \exists x Fx \rightarrow \exists y Gy$$

Let the domain M be $\{\text{Alice}, \text{Bob}\}$. We assume that Alice is a girl, and Bob is a boy. Interpret Fx as "x is a boy", and interpret Gx as "x is a square circle" (i.e. the extension of Gx is empty).

Since Alice is not a boy, she is not in the extension of the predicate Fx. Hence $Fx \rightarrow \exists y Gy$ is true (by negative paradox) of Alice. It follows that $\exists x(Fx \rightarrow \exists y Gy)$ is true. Since Bob is a boy, it follows that $\exists x Fx$ is true; but since there are no square circles, $\exists y Gy$ is false. Therefore, $\exists x Fx \rightarrow \exists y Gy$ is false.