

Logic Final Exam

Please note that this exam has two pages. Although you may take up to three hours to complete the exam, it was designed to take at most two hours. There are 50 possible points.

Proofs

Prove the following sequents. If you use a meta-rule (such as cut), then you must include a proof of the sequent that you cite. [6 points each; 18 points total]

1. $P \rightarrow \exists x Fx \vdash \exists x(P \rightarrow Fx)$
2. $\forall x \neg Gx \vdash \neg \exists x Gx$
3. $\forall x(\neg Rxx \rightarrow \exists y Ryx) \vdash \forall x \exists y Ryx$

Translation

Translate the following sentences into predicate logic notation. Please specify your translation scheme, for example “ $Rxy \equiv x \text{ respects } y$ ”. We have suggested symbols after each sentence. [3 points each; 12 points total]

1. There is a professor who is respected by any student who respects any professor at all. (Px, Sx, Rxy)
2. There is a professor who respects only those students who respect all professors. (Px, Sx, Rxy)
3. There is at most one professor who respects no students. ($Px, Sx, Rxy, =$)
4. Alice respects only Briana and Caroline. ($a, b, c, Rxy, =$)

Interpretations

1. Give an interpretation in which the following four sentences are all true. This interpretation must include not only a domain and an extension for the relation symbol R , but also an assignment of the names a, b, c to elements of the domain. [6 points]

$$\forall x(\neg Rxx \rightarrow Rax) \quad \exists x \neg Rxx \quad \forall x Rx b \quad \forall x (Rbx \leftrightarrow x = c)$$

2. Show that the following sequent is not truth-preserving by providing a countermodel. [6 points]

$$\forall x(\neg Rxx \wedge \exists y(Rxy \wedge \forall z(Ryz \rightarrow Rxz))) \vdash \forall y \exists x Rxy$$

Conceptual

1. Explain what's wrong with the following attempted proof. In particular, identify the “bad” lines (i.e. those lines where the truth of the dependencies does not ensure the truth of the sentence on the right), and explain why those lines are bad. [4 points]

1	(1)	$\forall x \exists y Rxy$	A
1	(2)	$\exists y Ray$	1 UE
3	(3)	Rab	A
3	(4)	$\forall x Rx b$	3 UI
3	(5)	$\exists y \forall x Rxy$	4 EI
1	(6)	$\exists y \forall x Rxy$	2,3,5 EE

2. Define what it means for a system of proof rules to be *sound* and *complete*. Give a system of rules for predicate logic that is complete but not sound. (Your system of rules may include rules that we learned this semester, but it may also have new rules that you've made up.) [4 points]

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