# Logic: Final Exam

Instructions: You have three hours to upload your answers from the time you open the exam. Please leave yourself at least ten minutes of cushion in case of technical problems, and if problems persist, please email your exam to your preceptor. You may use all notes, textbook, lectures, and even the internet, so long as you do not discuss the exam with other human beings. Please write your honor code pledge on your exam.

There are 22 points total. If your percentage on this exam is higher than your previous percentage in the course, then the exam will count for 30% of your course grade.

1. Translate the following sentences into predicate logic. Use $Rxy$ for "$x$ respects $y$", use $a$ for "Alice", $b$ for "Bob", and use the equality symbol "=" where appropriate. You may assume that quantifiers range over people, so you don't need any additional symbol for "$x$ is a person." [2 points each]

   (a) Alice respects someone other than Bob.

   (b) Bob is respected only by those people who respect everyone.

   (c) Some people respect at most one person besides themselves. [Although this sentence is a bit vague, we do not intend it to imply that these people respect themselves.]

2. Prove the following sequents. You may cut in any results that have been proven in the textbook, lectures, or previous psets (please cite specific page, lecture number, or pset). [4 points each]

   (a) $\vdash \exists x \forall y (Fy \to Fx)$

   (b) $\forall x(\neg Rxx \wedge \exists y(Rxy \wedge \forall z(Ryz \to Rxz))) \vdash \exists x \exists y \exists z(x \neq y \wedge (x \neq z \wedge y \neq z))$

3. Provide a counterexample to show that the following sequent is not valid. It will suffice to give an arrow diagram. [4 points]

$$\forall x Rxx,\ \forall x(\forall y Rxy \to \forall z Rzx) \vdash \forall x \forall y (Rxy \to Ryx)$$

4. True or false (justify your answer): there could be a correctly written proof with the following line fragments. [4 points]

$$\begin{array}{llll} 1 & (1) & \forall x \exists y (Fx \to Gy) & \text{A} \\ & \vdots & & \\ 1 & (n) & \forall x (Fx \to Ga) & \end{array}$$