Final Examination **PHI 201, Spring 2012**

Instructions: Please note that this exam has *two* pages. Write your name, preceptor's name, and pledge on the exam booklet; and write all of your answers in the exam booklet. You have three hours to complete the exam. When finished, place your exam booklet in the box at the front of the room, and leave quietly. There are 50 possible points in total.

Definitions [3 points each; 6 points total]

1. True or False (explain your answer): If it is *not* true that $A_1, \ldots, A_n \vdash B$, then it *is* true that $A_1, \ldots, A_n \vdash \neg B$.

2. Explain what's wrong with the following "proof" of the tautology $\exists x \forall y(Fy \to Fx)$. In particular, identify the "bad" lines (i.e. those lines where the sentence on the right is not a semantic consequence of the dependencies on the left).

1	(1)	Fa	A
	(2)	$Fa \to Fa$	1,1 CP
	(3)	$\forall y(Fy \to Fa)$	2 UI
	(4)	$\exists x \forall y(Fy \to Fx)$	3 EI

Translation [4 points each; 12 points total] Translate the following sentences into predicate logic notation. In each case, give a "dictionary" for the predicate and relation symbols that you use in the translation. (We have suggested some predicate and relation symbols at the end of each sentence.)

1. There is a professor who is respected by any student who respects any professor at all. (Px, Sx, Rxy)

2. There is a professor who respects only those students who respect her. (Px, Sx, Rxy)

3. There is no largest prime number. (Lxy, Px, Nx)

Proofs and Counterexamples [20 points total]

1. Prove the following tautology. (You may use SI only for things provable in propositional logic, and for the quantifier-negation equivalences.) [8 points]

$$\vdash \exists x \forall y(Fy \to Fx)$$

2. The two sentences "$\forall x \forall y(Rxy \to Ryx)$" and "$\forall x(Fx \leftrightarrow \exists y Rxy)$" together imply one of (a) and (b) below, but not the other. Find which is implied and show the implication by giving a proof (using SI only for propositional logic proofs or the quantifier-negation equivalences). Show that lack of implication in the other case by giving a suitable interpretation.

   (a) $\forall x Fx \to \exists x \forall y Rxy$

   (b) $\exists x \forall y Rxy \to \forall x Fx$

Metatheory [6 points each; 12 points total] Please complete two of the following three problems. If you give solutions to all three, please clearly designate which two problems you want to be graded.

1. State and prove the soundness of Conditional Proof (CP) relative to propositional logic interpretations (i.e. truth tables). That is, show that CP takes "good lines" to "good lines."

2. Let Σ be the set of sentences of propositional logic whose only atomic sentence is P. Show that for any consistent sentence A in Σ, either $P \vdash A$ or $\neg P \vdash A$. (You may cite any result that we established during the semester.)

3. Prove that the set $\{\wedge, \to\}$ of connectives is *not* truth-functionally complete.

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