Final Exam.

Please make sure that you have all three pages of the exam. Write your name and your preceptor's name on your exam booklet. You have three hours to complete the exam.

Short answer

1. Complete the following sentence: An argument with premises $\phi_1, \ldots, \phi_n$ and conclusion ψ is valid if...
2. If the algorithm for testing the consistency of simple monadic sentences (i.e., Algorithm B) is applied to a collection of six existential and two universal sentences, how many instances of each universal sentence will be produced?
3. Complete the following sentence: Sentence ϕ results from sentence ψ by substitution if...
4. True or False (explain your answer): If a sentence is consistent, then any substitution instance of that sentence is also consistent.
5. True or False (explain your answer): If $\phi_1, \ldots, \phi_n$ are inconsistent *propositional* calculus sentences, then there is a correctly written proof whose premises are $\phi_1, \ldots, \phi_n$ and whose conclusion is $P \,\&\, -P$.

Translation

Translate the following sentences into predicate calculus notation. Use the following dictionary.

$Mx \equiv x$ is male. $Pxy \equiv x$ is a parent of y.
$Axy \equiv x$ adores y.

(The domain of discourse is persons — you do not need a predicate symbol for "is a person." For the purposes of this problem, a "child" is anyone who has a parent.)

1. Some man who has a son adores him.
2. Everybody adores their own grandchildren.
3. Every woman adores her sisters' children.

4. No man adores children unless he has his own.

Proofs and Counterexamples

1. Consider the sentence "$(x)(y)[Qxy \leftrightarrow (z)(Rzx \to Rzy)]$".
   (a) Show by giving a proof that this sentence implies "$(x)Qxx$".
   (b) Give an interpretation that shows that the sentence does not imply "$(x)(y)(Qxy \to Qyx)$".
   (c) The sentence implies one of (i) and (ii) but not the other; give a proof to show the implication in the one case, and give an interpretation to show the lack of implication in the other:
      (i) $(\exists y)(x)Rxy \to (\exists y)(x)Qxy$
      (ii) $(\exists y)(x)Qxy \to (\exists y)(x)Rxy$

Semantics

1. Use the algorithm for pure monadic sentences (i.e., Algorithm C) to determine whether or not the following argument is valid. Show all steps. If it's invalid, show that by giving an interpretation.

   (1) $(\exists x)Gx \lor -(x)Fx$
   (2) $-(x) - Fx \to -(x)Fx$ $\quad / \ (\exists x)Fx \to (\exists x)Gx$

2. Give an interpretation with nonempty extension for R that shows that the first sentence does not imply the second.

$$(x)(z)[Rxz \to (\exists y)(Rxy \,\&\, Ryz)] \qquad (x)(y)(z)[(Rxy \,\&\, Ryz) \to Rxz]$$

Reflection

1. Find a sentence in prenex form that is equivalent to: $(x)((\exists y)Rxy \to (\exists z)Sxz)$.
2. The completeness proof for the predicate calculus relies on the fact that for any sentence ϕ in prenex form, there is an algorithm for producing instances of ϕ with arbitrary names $a_1, a_2, a_3, \ldots$. (The

quantifier-free sentences that result from applying this algorithm will be consistent if and only if ϕ is consistent.) Write out the first four stages of this algorithm applied to the sentence $(x)(\exists y)Rxy$.

For the following two problems, please give rigorous (but informal) arguments.

2. Show that logical implication is transitive. That is, if ϕ implies ψ and ψ implies χ, then ϕ implies χ.

3. State precisely what it means to say that the Predicate Calculus is "sound" and "complete." (i.e., state the Soundness and Completeness Theorems for the Predicate Calculus.) Prove the soundness of the rule of Universal Introduction (UI).