PHI 201 – Introductory Logic **Final Exam**

January 25, 2003

Please make sure that you have all **three** pages of the exam. Put your answers in the exam booklet, and write both your name and your preceptor's name on your booklet. You have 2.5 hours to complete the exam. There are 135 possible points.

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1. Define the following. (10 points total; 5 points each)
   (a) Valid argument
   (b) Tautology

2. Are the following true or false? In each case, *justify your answer.* (15 points total; 5 points each) You will get at most 2 points (out of 5) if you do not explain your answer.
   (a) If P is a sentence of the blocks language that is false in every block world (of Tarski's world), then $\neg\mathsf{P}$ is a first-order validity.
   (b) No matter what sentences P and Q are, the compound sentence $[\neg(\mathsf{P} \land \mathsf{Q}) \land \mathsf{Q}] \rightarrow \neg\mathsf{P}$ will be a logical truth.
   (c) Suppose that P is a (perhaps compound) sentence that is a first-order validity. Then P is true on at least one row of its truth table.

3. Are the following two sentences tautologically equivalent? Are they logically equivalent? Justify your answer fully. (10 points)

   $\neg\mathsf{Cube(a)} \land \neg\mathsf{Dodec(b)} \land \mathsf{a} = \mathsf{b}$ $\qquad\qquad$ $\mathsf{Tet(b)} \leftrightarrow \mathsf{Tet(a)}$

4. How many rows are there in the joint truth table for the two sentences in the previous problem? (4 points)

5. Translate the following sentences into FOL. Use the dictionary provided below. If a sentence is ambiguous, note this and provide what you think is the best possible translation. (25 points total; 5 points each)

$\mathsf{r} =$ Regine	$\mathsf{L(x)} \equiv \mathsf{x}$ is a letter
$\mathsf{s} =$ Søren	$\mathsf{P(x)} \equiv \mathsf{x}$ is a postal worker
$\mathsf{f(x)} \equiv$ father of x	$\mathsf{W(x, y)} \equiv \mathsf{x}$ was written by y
$\mathsf{A(x, y)} \equiv \mathsf{x}$ is addressed to y	$\mathsf{G(x, y, z)} \equiv \mathsf{x}$ gives y to z
$\mathsf{E(x)} \equiv \mathsf{x}$ is a person	

(a) Soren wrote a letter to Regine only if she wrote one to him.

(b) Søren wrote everybody's father a letter.

(c) Some postal workers give all letters to Regine.

(d) If Søren wrote every letter then no letter is addressed to Regine.

(e) Neither of the postal workers gave a letter to Søren.

6. Use the dictionary from the previous problem to translate the following FOL sentences into *colloquial* English. i.e., you should have no variables, and no phrases like "there exists." (10 points total; 5 points each)

(a) $\exists \mathsf{x}[\mathsf{L(x)} \land \mathsf{W(x, s)} \land \forall \mathsf{y}[(\mathsf{L(y)} \land \mathsf{W(y, s)}) \to \mathsf{x} = \mathsf{y}] \land \mathsf{A(x, r)}]$

(b) $\mathsf{P(f(r))} \to \exists \mathsf{x} \exists \mathsf{y}(\mathsf{L(x)} \land \mathsf{E(y)} \land \mathsf{G(f(r), x, y)})$

7. Is $\exists \mathsf{x} \exists \mathsf{y} \neg \mathsf{LeftOf(x, y)}$ a first-order consequence of $\exists \mathsf{x} \neg \mathsf{LeftOf(x, x)}$? If so, give a formal proof. If not, give a reinterpretation of LeftOf and an example where the premise is true and the conclusion is false. (10 points)

8. Prove that that the following arguments are valid using only the rules of $\mathcal{F}$. Do *not* cite "X-Con" (for any X) and do not cite "DeMorgans." (30 points; 10 points each)

(a)

$$\begin{array}{|l} \mathsf{A} \lor \mathsf{B} \\ \neg \mathsf{A} \lor \neg \mathsf{B} \\ \hline \neg(\mathsf{A} \leftrightarrow \mathsf{B}) \end{array}$$

(b)

$$\begin{array}{|l} \forall x(\mathsf{F}(x) \rightarrow \exists y \mathsf{G}(x,y)) \\ \forall x \forall y(\mathsf{G}(x,y) \rightarrow \mathsf{D}(x,y)) \\ \hline \forall x(\mathsf{F}(x) \rightarrow \exists y \mathsf{D}(x,y)) \end{array}$$

(c)

$$\begin{array}{|l} \forall x \exists y(\mathsf{A}(x) \land \mathsf{B}(y)) \\ \hline \exists y \forall x(\mathsf{A}(x) \land \mathsf{B}(y)) \end{array}$$

9. Show that the conclusion of the following argument is not a *tautological* consequence of its premises. (7 points)

$$\begin{array}{|l} \mathsf{Blond}(\mathsf{claire}) \land \forall x(\mathsf{Blond}(x) \rightarrow \mathsf{Loves}(\mathsf{max}, x)) \\ \forall x(\mathsf{Blond}(x) \rightarrow \mathsf{Loves}(\mathsf{max}, x)) \rightarrow \neg(\mathsf{Loves}(\mathsf{max}, \mathsf{claire}) \land \mathsf{Loves}(\mathsf{max}, \mathsf{max})) \\ \hline \neg\neg\neg\mathsf{Blond}(\mathsf{max}) \end{array}$$

10. Show that the conclusions of the following arguments are not *first-order* consequences of their premises. (14 points total; 7 points each)

(a)

$$\begin{array}{|l} \forall x(\mathsf{SameRow}(x,a) \rightarrow \mathsf{SameCol}(x,a)) \\ \exists x(\mathsf{Cube}(x) \land \mathsf{SameRow}(x,a)) \\ \hline \mathsf{Cube}(a) \end{array}$$

(b)

$$\begin{array}{|l} \exists x \mathsf{Cube}(x) \\ \forall x[\mathsf{Cube}(x) \rightarrow \exists y(\mathsf{Tet}(y) \land \mathsf{Larger}(x,y))] \\ \hline \exists x[\mathsf{Tet}(x) \land \forall y(\mathsf{Cube}(y) \rightarrow \mathsf{Larger}(x,y)] \end{array}$$

11. (Extra Credit; 2 points) What lesson did Bertrand Russell's turkey learn on Thanksgiving morning? (The correct answer is *not*: "Russell was British, and so he didn't celebrate Thanksgiving.")