Practice Final Exam

Short answer

1. State the Existential Elimination (EE) rule, along with each of its restrictions.
2. Complete the following sentence: propositional logic sentences ϕ and ψ are mutually consistent just in case ...
3. True or False (explain your answer): Suppose that ϕ and ψ are mutually consistent propositional logic sentences. Could there be a correctly written proof that begins with $\phi \wedge \psi$ and that ends with $\bot$?
4. Grade the following proof.

1	(1)	$p \vee q$	A
2	(2)	p	A
3	(3)	q	A
2,3	(4)	$p \wedge q$	2,3 $\wedge$I
2,3	(5)	p	4 $\wedge$E
1	(6)	p	1,2,2,3,5 $\vee$E

5. Grade the following proof.

1	(1)	$\neg p$	A
2	(2)	$\exists x(Fx \wedge \neg Fx)$	A
2	(3)	$\neg\neg p$	1,2 RAA
2	(4)	p	3 DN
	(5)	$\exists x(Fx \wedge \neg Fx) \to p$	2,4 CP

6. A "bad line" in a proof is a line where the sentence on the right is not a logical consequence of its dependencies. Identify all the bad lines in the previous two proofs.

Translation

Translate the following sentences into predicate logic notation. You may use the equals sign $=$ as well as the following relation symbols:

$Mx \equiv x$ is male $\quad Pxy \equiv x$ is a parent of $y \quad Axy \equiv x$ adores y

(The domain of quantification is persons — you do not need a predicate symbol for "is a person." For the purposes of this problem, a "child" is anyone who has a parent.)

1. Every man who has a son adores him.
2. Every man who has a daughter adores his daughter's mother.
3. Everybody adores their own grandchildren.
4. Every woman adores her brothers' children.
5. No man adores children unless he has his own.
6. Someone has no more than two children.

Proofs

Prove the following sequents.

1. $\neg(p \to q) \ \vdash \ (p \wedge \neg q)$
2. $\exists x(Fx \wedge \forall y(Gy \to Rxy)), \forall x(Fx \to \forall y(Hy \to \neg Rxy)) \ \vdash \ \forall x(Gx \to \neg Hx)$

Metatheory

1. Use proof by induction to show that the connective $\vee$ is not by itself truth-functionally complete (i.e. there is a truth-function that cannot be expressed using only $\vee$).
2. State precisely what it means to say that the propositional logic inference rules are *sound*. i.e. state the soundness theorem for the propositional calculus. Prove the soundness of Reductio ad Absurdum (RAA).
3. True or False (explain your answer): if ϕ is a propositional logic tautology, and ϕ' is a substitution instance of ϕ, then ϕ' is a tautology.
4. Give a substitution instance of the following sentence that is a tautology:

   $(p \wedge q) \vee (\neg p \wedge \neg q)$

   Explain precisely which substitutions you have performed.