Practice Final Exam

Short answer

1. State the Existential Elimination (EE) rule, along with each of its restrictions.
2. True or False (explain your answer): If $A_1, \ldots, A_n$ are inconsistent predicate logic sentences, then there is a correctly written proof whose premises are $A_1, \ldots, A_n$ and whose conclusion is $P \land \neg P$.
3. Complete the following sentence: Predicate logic sentences A and B are inconsistent just in case ... (Note: Please give the *semantic* definition that uses the concept of an "interpretation.")
4. Grade the following proof.

1	(1)	$p \lor q$	a
2	(2)	p	a
3	(3)	q	a
2,3	(4)	$p \land q$	2,3 $\land$I
2,3	(5)	p	4 $\land$E
1	(6)	p	1,2,2,3,5 $\lor$E

5. Grade the following proof.

1	(1)	$\neg p$	A
2	(2)	$\exists x(Fx \land \neg Fx)$	A
2	(3)	$\neg\neg p$	1,2 RAA
2	(4)	p	3 DN
	(5)	$\exists x(Fx \land \neg Fx) \to p$	2,4 CP

6. A "bad line" in a proof is a line where the sentence on the right is not a logical consequence of its dependencies. Identify all the bad lines in the previous two proofs.

Translation

Translate the following sentences into predicate logic notation. You may use the equals sign "=" as well as the following relation symbols:

$Mx \equiv x$ is male $\quad Pxy \equiv x$ is a parent of $y \quad Axy \equiv x$ adores y

(The domain of quantification is persons — you do not need a predicate symbol for "is a person." For the purposes of this problem, a "child" is anyone who has a parent.)

1. Every man who has a son adores him.
2. Every man who has a daughter adores his daughter's mother.
3. Everybody adores their own grandchildren.
4. Every woman adores her brothers' children.
5. No man adores children unless he has his own.
6. Someone has no more than two children.

Proofs and Counterexamples

1. Prove the following tautology using only basic rules of inference.

$$\vdash \neg(P \to Q) \leftrightarrow (P \wedge \neg Q)$$

2. Prove the validity of the following argument using only basic rules of inference.

$$\exists x(Fx \wedge \forall y(Gy \to Rxy)), \forall x(Fx \to \forall y(Hy \to \neg Rxy)) \vdash \forall x(Gx \to \neg Hx)$$

3. Consider the sentence "$\forall x \forall y[Qxy \leftrightarrow \forall z(Rzx \to Rzy)]$".
   (a) Show by giving a proof that this sentence implies "$\forall x Qxx$".
   (b) Give an interpretation that shows that the sentence does not imply "$\forall x \forall y(Qxy \to Qyx)$".
   (c) The sentence implies one of (i) and (ii) but not the other; give a proof to show the implication in the one case, and give an interpretation to show the lack of implication in the other:
      (i) $\exists y \forall x Rxy \to \exists y \forall x Qxy$
      (ii) $\exists y \forall x Qxy \to \exists y \forall x Rxy$

4. Determine whether or not the following argument is valid. If the argument is invalid, provide a counterexample interpretation. If the argument is valid, explain how you know that there is no counterexample.

$$\exists x Gx \vee \neg \forall x Fx \ , \ \neg \forall x \neg Fx \to \neg \forall x Fx \ \vdash \ \exists x Fx \to \exists x Gx$$

Metatheory

For the following problems, please give rigorous (but informal) arguments.

1. Use proof by induction to show that the connective "$\vee$" is not by itself truth-functionally complete (i.e. there is a truth-function that cannot be expressed using only "$\vee$").

2. State precisely what it means to say that the predicate logic inference rules are "sound" and "complete." (i.e. state the soundness and completeness theorems for the predicate calculus.) Prove the soundness of Reductio ad Absurdum (RAA).

3. Let's say that a "schminterpretation" of a predicate logic sentence is an interpretation whose domain of quantification has *at most two elements*; and let's say that a "schmautology" is a sentence that is true relative to all schminterpretations. Give an example of a schmautology that is not a tautology. Do not use the equality symbol "$=$".

Extra Credit

1. Prove that there is no pure monadic sentence that is true relative to all and only those interpretations whose domains have exactly n elements. [Hint: Show that if a pure monadic sentence A is true relative to an interpretation of size n (i.e., an interpretation whose domain has n elements), and $n < m$, then A is true relative to an interpretation of size m.]
2. Prove that if a pure monadic sentence is consistent, then it is true relative to some interpretation whose domain has a *finite* number of elements. [Hint: You may assume that Algorithm B is a reliable test of consistency for pure monadic sentences.]
3. Prove that the set $\{\neg, \leftrightarrow\}$ of connectives is not truth-functionally complete.